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ROUTING MIXED (SUBCRITICAL/SUPERCRITICAL) DAM-BREAK FLOWS USING NWS FLDWAV

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ABSTRACT

Routing unsteady "mixed" flows that change in time and/or space from subcritical to supercritical, or conversely can present numerical difficulties when using models based on the complete one-dimensional Saint-Venant equations of unsteady flow. The National Weather Service (NWS) has developed a model, FLDWAV, which has the capability to simulate mixed flows using a four-point implicit, nonlinear finite-difference technique for solving the Saint-Venant equations. Via an assortment of internal boundary conditions, the FLDWAV model can simulate time-dependent dam breaches, time-dependent gate controlled flows, assorted spillway flows, bridge/embankment overtopping flows, and levee overtopping and crevasse flows. FLDWAV can simulate flows that range from Newtonian (water) to non-Newtonian (mud/debris) which occur in a single waterway or multiple interconnected waterways in which sinuosity effects are considered, and flows that occur in expansive floodplains that may be compartmentalized by dikes and elevated roads. The mixed flow algorithm within FLDWAV is based on the concept of not requiring the solution of the Saint-Venant equations where the flow passes from supercritical to subcritical or conversely. Where and when this occurs, appropriate external boundary equations, i.e., critical flow or depth, are used; this divides the total routing reach into two or more sub-reaches wherein only subcritical or supercritical flow occurs. An example is presented illustrating the FLDWAV model's routing of a dam-break flood wave exhibiting the condition of mixed flow in a nonprismatic channel with an irregular bottom slope and further complicated by a moving hydraulic jump subjected to variable backwater conditions.

INTRODUCTION

Flood routing or unsteady flow simulation is an essential tool for flood forecasting and engineering design/analysis of hydraulic structures. A generalized flood routing model, FLDWAV, (Fread, 1985; Fread and Lewis, 1988) has been developed by the National Weather Service (NWS). It was developed to replace two widely used NWS models, DWOPER (Fread, 1978) and DAMBRK (Fread, 1985, 1988), since it will utilize their combined unsteady flow simulation capabilities, as well as provide new hydraulic simulation features and improved user-friendly data input. The model can be used by hydrologists/engineers for a wide range of unsteady flow applications including dam-breach analysis and inundation mapping for sunny-day piping failures or overtopping failures due to PMF reservoir inflows including the complexities associated with failure of two or more dams sequentially located along a watercourse.

This paper presents a description of the governing equations of the FLDWAV model, focusing on the capability of the FLDWAV model to route unsteady "mixed" flows that change in time and/or

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space from subcritical to supercritical, or conversely. To avoid numerical difficulties, these need particular attention and algorithmic features. An example is presented of the FLDWAV model's routing of a dam-break flood exhibiting the condition of mixed flow in a nonprismatic channel with an irregular bottom slope and complicated by a moving hydraulic jump subjected to variable backwater from a downstream tributary inflow.

GOVERNING EQUATIONS

The governing equations of the FLDWAV model are: (1) expanded one-dimensional equations of unsteady flow originally derived by Saint-Venant; (2) an assortment of internal boundary equations of flow through one or more flow control structures located along the main-stem river and/or its tributaries; and (3) external boundary equations of known upstream/downstream discharges or water elevations which vary either with time or each other.

Expanded Saint-Venant Equations:

An expanded form of the Saint-Venant equations of conservation of mass and momentum consist of the following:

$$\partial Q/\partial x + \partial s_c(A+A_o)/\partial t - q = 0 \tag{1}$$

in which Q is discharge (flow); A is wetted active cross-sectional area; A_o is wetted inactive off-channel (dead) storage area associated with topographical embayments or tributaries; s_c is a depth-dependent channel sinuosity coefficient (Delong, 1986; Fread, 1988); q is lateral flow (inflow is positive, outflow is negative); t is time; and x is distance measured along the mean flow-path of the floodplain. The conservation of momentum equation is:

$$\partial(s_m Q)/\partial t + \partial(\beta Q^2/A)/\partial x + gA(\partial h/\partial x + S_f + S_e + S_i) + L = 0 \tag{2}$$

in which s_m is another depth-dependent sinuosity coefficient, g is the gravity acceleration constant; h is the water surface elevation; L is the momentum effect of lateral flows ($L = -qv_x$ for lateral inflow, where v_x is the lateral inflow velocity in the x-direction; $L = -q\ (Q/(2A)$ for seepage lateral outflows; $L = -q\ Q/A$ for bulk lateral outflows); S_f is the boundary friction slope, i.e., $S_f = |Q|Q/K^2$ in which K is the total conveyance determined by summing conveyances of the left/right floodplains and channel in which the channel conveyance is modified by the factor, $1/s_m^{1/2}$, and all conveyances are determined automatically from the data input of topwidth/Manning n versus elevation tables for cross sections of the channel and left/right floodplains; S_e is the expansion/contraction slope, i.e., $S_e = k_e/(2g) \cdot \partial(Q/A)^2/\partial x$ where k_e is the expansion/contraction loss coefficient; β is the momentum coefficient for non-uniform velocity distribution and is internally computed from the conveyances and areas of the channel and left/right floodplains and S_i is the internal viscous dissipation slope for non-Newtonian (mud/debris) flows (Fread, 1988), i.e.,

$$S_i = \kappa/\gamma[(b+2)Q/(AD^{b+1}) + (b+2)/(2D^b)\ (\tau_o/\kappa)^b]^{1/b} \tag{3}$$

in which D=A/B where B is the wetted topwidth; κ is the apparent fluid viscosity; γ is the fluid's unit weight; τ_o is the initial shear strength of the fluid; and $b = 1/m$ where m is the exponent of a power function that represents the fluid's stress (τ_s)-rate of strain (dv/dy) relation, i.e., $\tau_s = \tau_o + \kappa(dv/dy)^m$ in which v and y are the flow velocity and depth, respectively.

Internal Boundary Equations:

Locations along the main-stem and/or tributaries where the flow is rapidly varied in space and Eqs. (1-2) are not applicable, e.g. dams, bridges/road-embankments, waterfalls, short steep rapids, weirs, etc. These locations require the following internal boundary equations in lieu of Eqs. (1-2):

$$Q_i - Q_{i+1} = 0 \tag{4}$$

$$Q_i = f(h_i, h_{i+1}, \text{properties of control structure}) \tag{5}$$

in which the subscripts i and i+1 indicate cross sections just upstream and downstream of the structure, respectively. For a bridge, Eq. (5) becomes:

$$Q = \sqrt{2g}\; C_b A_b\,(h_i - h_{i+1} + v^2/2g - \Delta h_f)^{1/2} + C_e L_e K_e (h_i - h_e)^{3/2} \tag{6}$$

in which C_b is the coefficient of flow through the bridge, A_b is the wetted cross-sectional area of the bridge opening, $v = Q/A$, Δh_f is the head loss through the bridge, C_e is the coefficient of discharge for flow over the embankment, L_e is the length of the road embankment, h_e is the elevation of the embankment crest, and K_e is a broad-crested weir submergence correction, i.e., $K_e = 1\text{-}23.8\,[(h_{i+1} - h_e)/(h_i - h_e) - 0.67]^3$. If the flow structure is a dam, Eq. (5) becomes:

$$Q = K_s C_s L_s (h_i - h_s)^{3/2} + \sqrt{2g}\; C_g A_g (h_i - h_g)^{1/2} + K_d C_d L_d (h_i - h_d)^{3/2} + Q_t + Q_{br} = 0 \tag{7}$$

in which K_s, C_s, L_s, and h_s are the uncontrolled spillway's submergence correction factor, coefficient of discharge, length of spillway, and crest elevation, respectively; K_d, C_d, L_d, and h_d are similar properties of the crest of the dam; C_g, A_g, and h_g are the coefficient of discharge, area, and height of opening of a fixed or time-dependent moveable gate spillway; Q_t is a constant or time-dependent turbine discharge; and Q_{br} is a time-dependent dam breach flow (Fread, 1977), i.e.,

$$Q_{br} = C_v K_b [3.1\; b_i (h_i - h_b)^{3/2} + 2.45\; z\; (h - h_b)^{5/2}] \tag{8}$$

in which b_i is the known time-dependent bottom width of the breach, h_i is the known time-dependent bottom elevation of the breach, z is the side slope of the breach (1: vertical to z: horizontal), C_v is a velocity of approach correction factor, and K_b is a broad-crested weir submergence correction factor similar to K_e in Eq. (6). Breach properties may be determined from empirical statistical relations (Fread, 1988) or from breach simulation models, e.g. (Fread, 1984).

External Boundary Equations:

External boundary equations used at the upstream and downstream extremities of the waterway may be a specified time series of discharge (a discharge hydrograph) or water elevation as in the case of a lake level or estuarial tidal fluctuation. At the downstream extremity, the boundary equation can be Eq. (7), an empirical rating of h and Q, or a channel control, loop-rating based on the Manning equation in which S (the dynamic energy slope) is approximated by:

$$S = (h_{N-1} - h_N)/\Delta x - (Q^{t+\Delta t} - Q)/(gA\;\Delta t) - [(Q^2/A)_N - (Q^2/A)_{N-1}]/(gA\;\Delta x) \tag{9}$$

in which Δx is the distance between the last two cross sections at the downstream boundary.

Solution Technique:

In FLDWAV, the Saint-Venant Eqs. (1-2) are solved by a weighted four-point nonlinear implicit finite-difference technique as described by (Fread, 1985). Substitution of appropriate simple algebraic approximations for the derivative and non-derivative terms in Eqs. (1-2) result in two nonlinear algebraic equations for each Δx reach between specified cross sections which, when combined with the external boundary equations and any necessary internal boundary equations, may be solved by an iterative quadratic solution technique (Newton-Raphson) along with an efficient, compact, quad-diagonal Gaussian elimination matrix solution technique. Initial conditions required at t=0 are automatically obtained via a steady flow backwater solution. A river system consisting of a main-stem river and one or more principal tributaries is efficiently solved using an iterative relaxation method (Fread, 1985). If the river consists of bifurcations such as islands and/or complex dendritic systems with tributaries connected to tributaries, etc., a network solution technique is used (Fread, 1985), wherein three internal boundary equations conserve mass and momentum at each confluence. Solution of this system of algebraic equations requires another special sparse-matrix Gaussian elimination technique.

Special Features:

The FLDWAV model has several features including: (1) a subcritical/supercritical mixed-flow solution algorithm (details of which will follow), levee overtopping/floodplain interactions, automatic calibration (Fread, 1985), combined free surface/pressurized flow capabilities, and automatic selection of computational Δx and Δt steps.

SUBCRITICAL/SUPERCRITICAL MIXED FLOW ALGORITHM

The mixed flow algorithm automatically subdivides the total routing reach into sub-reaches wherein only subcritical or supercritical flow occurs. The transition locations where flow changes from subcritical to supercritical or vice versa are treated as boundary conditions thus avoiding the application of the Saint-Venant equations to the transition flow and subsequent numerical solution difficulties. The mixed-flow algorithm has two components, one for obtaining the initial condition of discharge and water elevation at $t=0$ and another which functions during the unsteady flow solution.

The initial condition component obtains the water elevations by the following algorithm: (1) normal and critical depths are obtained for each section -- the section is designated subcritical if normal depth is greater than critical depth, or it is designated supercritical if normal is less than critical after a check is made to see if upstream elevations created by a dam may drown-out upstream supercritical depths; (2) commencing at the downstream boundary, a backwater solution proceeds from a known elevation (dependent on the downstream boundary condition at $t=0$) in an upstream direction until supercritical flow occurs or if supercritical flow occurs at the downstream boundary, the computations proceed in the downstream direction from the normal depth at the upstream-most section of all contiguous sections having supercritical flow; (3) when internal boundaries, such as a dam, are encountered, the specified water elevations occurring at $t=0$ for each reservoir are used for the backwater solution or if a bridge is encountered, Eq. (6), is solved iteratively until the correct value of h_i is determined from known values of Q_i and h_{i+1}.

The unsteady flow component groups contiguous sections with a Froude number less than or equal to 0.95 into subcritical sub-reaches and those with a Froude number greater than or equal to 1.05 into supercritical sub-reaches. Those sections with Froude numbers between 0.95 and 1.05 are considered critical sections. However, critical sections that are surrounded by subcritical sections are grouped with a subcritical sub-reach, while critical sections amongst supercritical sections are grouped with a supercritical sub-reach. The upstream and downstream limits of the subcritical/supercritical reaches are noted and used to determine the range over which the Saint-Venant finite-difference equations are applied. During a Δt time step, the solution commences with the most upstream sub-reach and proceeds sub-reach by sub-reach in the downstream direction. The upstream and downstream boundary conditions for each sub-reach are selected according to the following algorithm: (1) if the most upstream reach is subcritical, the upstream boundary is $Q(t)$ and the downstream boundary is the critical flow equation since flow must pass through critical when the next downstream sub-reach is supercritical: (2) if the most upstream reach is supercritical, the upstream boundary is $Q(t)$ and a rating curve $Q(h)$, and a downstream boundary is not required for the supercritical reach since flow disturbances created downstream of the supercritical reach cannot propagate upstream into the supercritical reach; (3) if an inner sub-reach is supercritical, the following equations are used for the two upstream boundary equations:

$$Q_1 = Q'(t) \tag{10}$$

$$h_1 = h'(t) \tag{11}$$

in which $Q'(t)$ is the most recently computed flow at the last cross section of the upstream subcritical sub-reach and $h'(t)$ is the computed critical water surface elevation of the downstream most cross section of the upstream subcritical sub-reach; (4) if an inner sub-reach is subcritical, Eq. (10) is used for the upstream boundary in which $Q'(t)$ represents the computed flow at the last section of the upstream supercritical sub-reach and the critical flow equation is used as the downstream boundary; (5) if the most downstream sub-reach is subcritical, Eq. (10) is used for the upstream boundary condition and the downstream boundary condition is user-specified as previously described in "External Boundary Equations"; (6) if the most downstream sub-reach is supercritical, Eqs. (10-11) are used as the upstream boundary equations, and no downstream boundary is required. To account for the possible upstream movement of the hydraulic jump, the following procedure is utilized before advancing to the next time step: (1) the subcritical elevation (h_e) is extrapolated to the adjacent upstream supercritical section; (2) the sequent water surface elevation of the adjacent upstream supercritical section is iteratively computed via the bi-section method of solving the following sequent elevation equation:

$$Q^2/(gA) + \bar{z}A - Q^2/(gA_s) - \bar{z}_s A_s = 0 \qquad (12)$$

in which $\bar{z}$ is the distance from the water surface to the center of gravity of the wetted cross section, A is the wetted area, Q is the computed flow at the section, and the subscript (s) represents variables associated with the sequent elevation h_s while the variables with no superscript are associated with the supercritical elevation; (3) if the sequent elevation h_s is greater that the extrapolated elevation (h_e), the jump is not moved upstream; however, if $h_s \leq h_e$, the jump is moved upstream section by section until $h_s > h_e$. To account for the possibility of the jump moving downstream (if it did not move upstream), the following procedure is utilized before advancing to the next time step: (1) starting at the most upstream section of the subcritical sub-reach, the supercritical elevation is computed using a downwater steady flow equation similar to a backwater equation, and its sequent elevation (h_s) is computed by applying the iterative bi-section method to Eq. (12); (2) using the most recently computed subcritical elevation (h), if $h \geq h_s$, the jump is not moved downstream; however, if $h < h_s$, the jump is moved downstream section by section until $h \geq h_s$. Possible jump movements are not computed for cases where the flow is essentially critical in several adjacent reaches, since this can cause some numerical difficulties.

Smaller computational distance steps (Δx) are required in the vicinity of the transition reaches between subcritical and supercritical flow. This is particularly required both upstream and downstream of a critical flow section to avoid numerical difficulties. Smaller Δx reaches also will enable more accurate location of hydraulic jumps. A very convenient feature for specifying any size computational distance step utilizing interpolated cross sections is available within FLDWAV.

APPLICATION

The FLDWAV model with dam-break flood generation and mixed subcritical/supercritical flow routing capabilities is applied to the following realistic hypothetical case.

A 95-ft high earth dam is subjected to 1 ft of overtopping water which precipitates a dam-breach. The breach is trapezoidal-shaped with an average width of 315 ft, a bottom width of 250 ft, and a side slope of 1 vertical: 0.67 horizontal; the breach requires 0.60 hours to completely form. The initial flow emanating from the dam is 6000 cfs. The channel downstream of the dam is nonprismatic and is represented with 15 selected cross sections irregularly spaced along the 12-mile downstream reach. The widths vary from cross-section to cross-section along the channel, e.g., the maximum channel width at mile 1.0 is 1000 ft while the next section downstream at mile 1.67 has a maximum width of 350 ft. Also, the channel bottom slope, shown in Fig. 1, varies from steeper slopes upstream ranging between 96 and 39 ft/mile to that of the lower end of the 12-mile reach with a slope of 2 ft/mile. A hydraulic jump occurs at mile 5.0 where the slope changes from 39 ft/mile to 2 ft/mile. A tributary enters immediately below this location with a peak flow of 50,000 cfs which subjects the hydraulic jump to variable backwater effects and causes it to move upstream approximately 1000 ft.

The maximum water surface elevations provided by the dam-breach flood are shown in Fig. 1. The dam-breach flood has a peak discharge of 95,000 cfs as shown in Fig. 2. The hydrographs produced by the FLDWAV model for 0.01, 2.65, 4.28, 5.0, 5.1, 9.0, and 12.0 miles below the dam are also shown in Fig. 2. Both supercritical and subcritical flows occur upstream of mile 5.0 with subcritical flow prevailing at all locations further downstream. During high flows, the reach upstream of mile 5.0 is entirely supercritical while the downstream portion remains subcritical.

Froude numbers range from 0.23 to 1.65. The computational time and distance steps were 0.03 hr and 0.08 to 0.35 miles, respectively. As shown in Fig. 2, the hydrograph peak attenuates as the dam-breach flood wave propagates downstream until the added inflow of the tributary flood causes an increase in the peak discharge at locations below mile 5.0. The increased peak then attenuates dramatically as the combined flood wave propagates further downstream through the flat portion of the downstream channel.

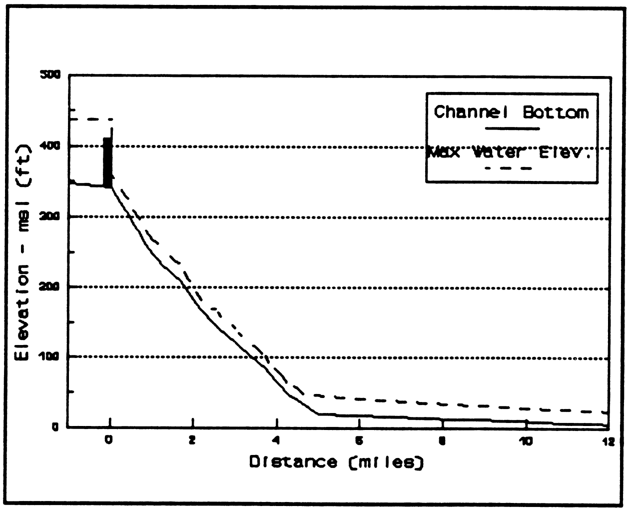

Fig. 1 Profile Downstream of Dam

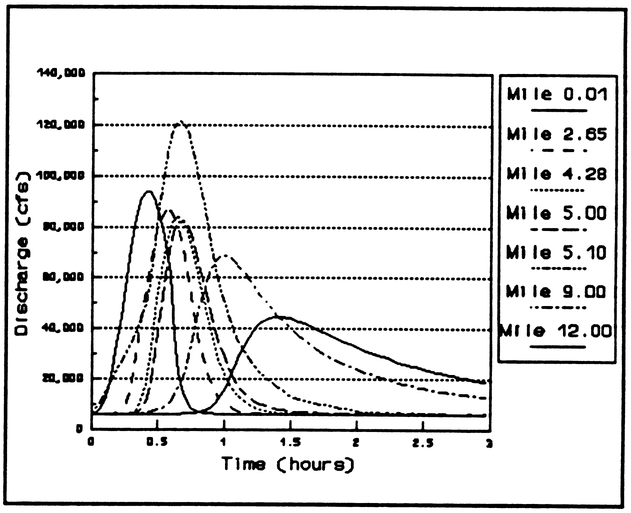

Fig. 2 Selected Hydrographs Downstream of Dam

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